

MATH 161: Quiz 2

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

1. Rationalize and completely simplify:

$$\frac{\overbrace{\sqrt{x} - \sqrt{x+h}}^{A - B}}{h} \cdot \frac{\overbrace{\sqrt{x} + \sqrt{x+h}}^{A + B}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \frac{\overbrace{x - (x+h)}^{A^2 - B^2}}{h(\sqrt{x} + \sqrt{x+h})} = \frac{x - x - h}{h(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-h}{h(\sqrt{x} + \sqrt{x+h})} \stackrel{\text{frac}}{\text{low 5}} \boxed{\frac{-1}{\sqrt{x} + \sqrt{x+h}}}$$

2. Suppose $f(x) = x - x^2$. Evaluate and completely simplify/factor the following:

(a) $f(-1) = (-1) - (-1)^2 = -1 - 1 = \boxed{-2}$

(b) $f(x+h) - f(x)$

$$= \overbrace{(x+h) - (x+h)^2}^{f(x+h)} - \overbrace{(x - x^2)}^{f(x)}$$

$$= x+h - (x^2 + 2xh + h^2) - x + x^2$$

$$= x+h - x^2 - 2xh - h^2 - x + x^2$$

$$= h - 2xh - h^2 = h \cdot (1 - 2xh - h)$$

3. Given the problem

$$\frac{x^{-1} + y^{-1}}{2}$$

(a) What type of fraction structure is this called?

Compound fraction

(b) Now fully simplify this expression.

for a compound fraction, this means no nested fractions.

$$\frac{x^{-1} + y^{-1}}{2} \stackrel{\text{def of } a^{-n}}{=} \frac{\frac{1}{x} + \frac{1}{y}}{2} \stackrel{\text{treat numerator as subproblem}}{=} \frac{\frac{y}{y} \cdot \frac{1}{x} + \frac{1}{y} \cdot \frac{x}{x}}{2}$$

$$\stackrel{\text{frac law (1)}}{=} \frac{\frac{y}{xy} + \frac{x}{xy}}{2} \stackrel{\text{frac law (2)}}{=} \frac{\frac{x+y}{xy}}{2}$$

$$\stackrel{\text{frac law (2)}}{=} \frac{x+y}{xy} \cdot \frac{1}{2} = \boxed{\frac{x+y}{2xy}}$$

4. If

$$f(x) = 1 - x \quad g(x) = 1 - x^2$$

evaluate and fully simplify the function $(f \circ g)(x)$.

$$(f \circ g)(x) \stackrel{\text{def}}{=} f(g(x))$$

$$= f(1 - x^2)$$

$$= 1 - (1 - x^2)$$

$$= 1 - 1 + x^2$$

$$= \boxed{x^2}$$